Corrigé de la planche 12

Exercice 1: $dsolve(\{y''(x) - y(x) = 0, y(0) = 2, y'(0) = 0\});$

$$y(x) = e^{x} + e^{-x} \tag{1}$$

$dsolve(\{y''(x) - y(x) = \exp(2\cdot x), y(0) = 0, y'(0) = 0\});$

$$y(x) = \frac{1}{6}\, e^{-x} - \frac{1}{2}\, e^{x} + \frac{1}{3}\, e^{2x} \tag{2}$$

Exercice 2: $dsolve(\{y''(x) - 4\cdot y'(x) + 4\cdot y(x) = 0, y(0) = 0, y'(0) = 1\});$

$$y(x) = e^{2x}\, x \tag{3}$$

$dsolve(\{y''(x) - 4\cdot y'(x) + 4\cdot y(x) = x\cdot\exp(4\cdot x), y(0) = 0, y'(0) = 0\});$

$$y(x) = \frac{1}{4}\, e^{2x} + \frac{1}{4}\, e^{2x} x + \frac{1}{4}\, (-1 + x)\, e^{4x} \tag{4}$$

Exercice 3: $dsolve(\{y''(x) + y'(x) + y(x) = 0, y(0) = 1, y'(0) = 1\});$

$$y(x) = e^{-\frac{1}{2}x} \sin\left(\frac{1}{2}\sqrt{3}\, x\right)\sqrt{3} + e^{-\frac{1}{2}x} \cos\left(\frac{1}{2}\sqrt{3}\, x\right) \tag{5}$$

$dsolve\left(\left\{y''(x) + y'(x) + y(x) = \cos\left(\frac{\mathrm{sqrt}(3)}{2}\cdot x\right), y(0) = 1, y'(0) = 1\right\}\right);$

$$y(x) = \frac{11}{39}\, e^{-\frac{1}{2}x} \sin\left(\frac{1}{2}\sqrt{3}\, x\right)\sqrt{3} + \frac{9}{13}\, e^{-\frac{1}{2}x} \cos\left(\frac{1}{2}\sqrt{3}\, x\right) + \frac{8}{13} \sin\left(\frac{1}{2}\sqrt{3}\, x\right)\sqrt{3} + \frac{4}{13} \cos\left(\frac{1}{2}\sqrt{3}\, x\right) \tag{6}$$

Exercice 4: $dsolve(\{y''(x) + 4\cdot y(x) = 0, y(0) = 0, y'(0) = 0\});$

$$y(x) = 0 \tag{7}$$

$dsolve(\{y''(x) + 4\cdot y(x) = 0, y(1) = 1, y'(1) = 2\});$

$$y(x) = \frac{(\sin(2) + \cos(2))\sin(2x)}{\sin(2)^2 + \cos(2)^2} - \frac{(-\cos(2) + \sin(2))\cos(2x)}{\sin(2)^2 + \cos(2)^2} \tag{8}$$

$dsolve(\{y''(x) + 4\cdot y(x) = \cos(2\cdot x), y(0) = 0, y'(0) = 0\});$

$$y(x) = \frac{1}{4}\sin(2x)\, x \tag{9}$$

Exercice 5: $dsolve\left(y''(x) - 2\cdot y'(x) + y(x) = 2\cdot x^2 - 8\cdot x + 5\right);$

$$y(x) = e^{x}\, \_C2 + e^{x}\, x\, \_C1 + 1 + 2\, x^2 \tag{10}$$

$dsolve\left(y''(x) - 3\cdot y'(x) + 2\cdot y(x) = x^2\right);$

$$y(x) = \frac{7}{4} + \frac{3}{2}\, x + \frac{1}{2}\, x^2 + \_C1\, e^{2x} + e^{x}\, \_C2 \tag{11}$$

Exercice 6: $dsolve\left(y''(x) + y'(x) = 1 + x^2\right);$

$$y(x) = -x^2 + \frac{1}{3}x^3 - e^{-x}\_C1 + 3x + \_C2 \tag{12}$$

$dsolve\left(y''(x) + 3\cdot y'(x) + 2\cdot y(x) = \left(x^2+1\right)\exp(-x)\right);$

$$y(x) = \left(-x^2 + \frac{1}{3}x^3 - e^{-x}\_C1 + 3x + \_C2\right)e^{-x} \tag{13}$$

Exercice 7: $dsolve(y''(x) - 4\,y(x) = x\cdot\exp(2\cdot x));$

$$y(x) = e^{2x}\_C2 + e^{-2x}\_C1 + \frac{1}{16}e^{2x}x\,(-1+2x) \tag{14}$$

$dsolve(y''(x) - 2\cdot y'(x) + y(x) = x\cdot\exp(x));$

$$y(x) = e^{x}\_C2 + e^{x}x\_C1 + \frac{1}{6}x^3 e^{x} \tag{15}$$

Exercice 8: $dsolve\left(y''(x) + w^2\cdot y(x) = 0\right);$

$$y(x) = \_C1\sin(wx) + \_C2\cos(wx) \tag{16}$$

$dsolve\left(\left\{y''(x) + k^2\cdot y(x) = 0, y(0) = 0\right\}\right);$

$$y(x) = \_C1\sin(kx) \tag{17}$$

Exercice 9: $dsolve(y''(t) - y(t) = 0);$

$$y(t) = \_C1\,e^{-t} + \_C2\,e^{t} \tag{18}$$

$dsolve(2\cdot y''(t) + 3\cdot y'(t) + 4\cdot y(t) = 0);$

$$y(t) = \_C1\,e^{-\frac{3}{4}t}\sin\left(\frac{1}{4}\sqrt{23}\,t\right) + \_C2\,e^{-\frac{3}{4}t}\cos\left(\frac{1}{4}\sqrt{23}\,t\right) \tag{19}$$

$dsolve(y''(t) + 2\cdot y'(t) + 3\cdot y(t) = 0)$

$$y(t) = \_C1\,e^{-t}\sin\left(\sqrt{2}\,t\right) + \_C2\,e^{-t}\cos\left(\sqrt{2}\,t\right) \tag{20}$$

$dsolve(y''(t) - 8\cdot y'(t) + 16\cdot y(t) = 0)$

$$y(t) = \_C1\,e^{4t} + \_C2\,e^{4t}t \tag{21}$$

Exercice 10: $dsolve(\{y''(t) - 3\cdot y'(t) - 4\cdot y(t) = 0, y(0) = 1, y'(0) = 0\});$

$$y(t) = \frac{1}{5}e^{4t} + \frac{4}{5}e^{-t} \tag{22}$$

$dsolve(\{y''(t) + y'(t) + 2\cdot y(t) = 0, y(0) = 1, y'(0) = -2\});$

$$y(t) = -\frac{3}{7}\sqrt{7}\,e^{-\frac{1}{2}t}\sin\left(\frac{1}{2}\sqrt{7}\,t\right) + e^{-\frac{1}{2}t}\cos\left(\frac{1}{2}\sqrt{7}\,t\right) \tag{23}$$

$dsolve(\{4\cdot y''(t) + 4\cdot y'(t) + y(t) = 0, y(0) = 0, y'(0) = 3\})$

$$y(t) = 3\,e^{-\frac{1}{2}t}t \tag{24}$$